

Super-group field cosmology in Batalin-Vilkovisky formulation

Sudhaker Upadhyay*

Centre for Theoretical Studies, Indian Institute of Technology Kharagpur, Kharagpur, 721 302, WB, India

In this paper, we study the third quantized super-group field cosmology, a model in multiverse scenario, in Batalin-Vilkovisky (BV) formulation. Further, we propose the superfield/super-antifield dependent BRST symmetry transformations. Within this formulation, we establish connection between the two different solutions of the quantum master equation within the BV formulation.

I. INTRODUCTION

The construction of a consistent theory of quantum gravity continues to be one of the major open problems in fundamental physics. Such a theory is very essential for the understanding of fundamental issues such as the origin of the Universe, the final evaporation of black holes, and the structure of space and time. Several approaches to quantum gravity have been developed, with a remarkable convergence [1]. The loop quantum gravity, a background-independent approach, is one of the powerful candidates quantizing gravity in mathematically rigorous and in non-perturbative way [2, 3]. The development started with the introduction of Ashtekar-Barbero variables, the densitized triad and the Ashtekar connection [4–9]. However, Loop quantum cosmology is the result of applying principles of loop quantum gravity to cosmological settings. The ensuing framework of loop quantum cosmology was introduced by Martin Bojowald [10]. The mathematical structure of loop quantum cosmology is presented in Ref. [11]. Loop quantum cosmology is constructed via a truncation of the classical phase space of general relativity to spatially homogeneous situations, which is then quantized by using the methods and results of loop quantum gravity. The quantization of geometric operators are thereby transferred to the truncated models.

However, the group field theories have been proposed as a kind of second quantization of canonical loop quantum gravity, in the sense that its canonical wave function turns into a dynamical (quantized) field [12, 13]. The group field theories are basically described by the field theories on group manifolds (or their Lie algebras) which provide a background-independent third quantized formalism for gravity in any dimensions and signature [14, 15]. In the group field theories, both the geometry and the topology are dynamical. The Feynman diagrams of such theories have an interpretation of the spacetimes and therefore the quantum amplitudes for these diagrams can be interpreted as algebraic realization of a path integral description of gravity [16, 17].

The topology changing processes can not be analyzed by second quantization approach. This brings group field theory into the conceptual framework of “third quantization”, for a rather appealing idea [18–25]. The third quantization is a field theory on the space of geometries, rather than spacetime, which also allows for a dynamical description of topology change [26]. Remarkably, the third quantization of loop quantum gravity leads to the group field theory [27–30]. The Wheeler-De Witt (mini-superspace) approximations of the group field theory results in the group field cosmology [31–38].

On the other hand, supersymmetry is an attractive concept whose basic feature is a transformation which relates bosons to fermions and vice-versa [39, 40]. One of its more significant features is that the presence of local supersymmetry naturally entails spacetime to be curved. The promotion of supersymmetric theory to a gauge symmetry has resulted in supergravity [39]. At large scales, supergravity permits to make the same predictions for classical tests as general relativity. The supersymmetry has also been testified as a prominent candidate for the dark matter [41]. The supersymmetrization of group field

*Electronic address: sudhakerupadhyay@gmail.com

cosmology has been made recently which is known as super-group field cosmology [42, 43]. Remarkably, the super-group field cosmology turns out to be gauge invariant and according to standard quantization principles a gauge symmetric models must be quantized after fixing a gauge as they possesses some spurious degrees of freedom. Hence to get rid of spurious degrees of freedom one should fix the gauge in case of super-group field cosmology. Such gauge-fixing adds the Faddeev-Popov ghosts in the void functional (the vacuum functional of third quantization) of the theory which helps in defining the physical Hilbert state of the effective theory. The fermionic rigid BRST transformation and thus unitarity of super-group field cosmology has been studied recently [44]. The Slavnov-Taylor identity and renormalizability of the theory has also been proved [43]. The cosmological Slavnov-Taylor identity of the Einstein-Hilbert action coupled to a single inflation field is derived recently in the Arnowitt-Deser-Misner formalism [45]. BRST algebra for the mixed Weyl-diffeomorphism residual symmetry is derived in Ref. [46]. The BRST symmetry gets relevance in many more contexts also [47].

The BV formalism [48–52], also known as the field/antifield and the BRST-BV approaches, is one of the most powerful techniques in the study gauge field theories which deals with very general gauge theories, including those with open or reducible gauge symmetry algebras. The BV method of quantization provides a convenient way of analysing the possible violations of symmetries [49]. Basically, it is used to perform the gauge-fixing in quantum field theory, however, it was also applied to other problems like analysing possible deformations of the action and anomalies. The BV approach is a successful for studying the manifestly Lorentz invariant formulation of the string theory [53]. Utilizing variational tricomplex, a covariant procedure for deriving the classical BRST charge from a given BV master action is proposed recently [54]. Using BRST-BV formulation of relativistic dynamics, arbitrary spin massless and massive field propagating in flat space and arbitrary spin massless fields propagating in AdS space are studied [55]. The BV formalism for the theory of super-group field cosmology has not studied yet. We try to write the quantum master equation for extended quantum action. The quantum master equation gives relation between different sets of greens functions and vertices.

The finite field-dependent BRST (FFBRST) transformation has been investigated originally in [56]. Further it has been found many implications in the diverse gauge theories [56–76]. For example, more recently, the gauge-fixing and ghost terms corresponding to Landau and maximal Abelian gauge have been rendered for the Cho-Faddeev-Niemi decomposed $SU(2)$ theory using FFBRST transformation [66]. However, the connection between linear and non-linear gauges for perturbative quantum gravity at both classical and quantum level has been established through FFBRST formulation [67, 68]. The quantum gauge freedom studied by gaugeon formalism has also been addressed for quantum gravity [69] as well as for Higgs model [70] utilizing FFBRST technique. The FFBRST transformations get relevance for the lattice gauge theory [73] and the relativistic point particle model [72]. With the help of FFBRST transformations it has also been proved that the problems associated with Virasoro constraints in worldsheet gravity are the gauge artifact [75]. Recently, Regge-Teitelboim cosmological model is quantized from the FFBRST point of view [77]. The FFBRST symmetries has also been derived in Friedmann-Robertson-Walker cosmological models [78]. Moshin and Reshetnyak in Ref. [79] systematically incorporates the case of BRST-antiBRST symmetry in Yang-Mills theories within the context of finite transformations that deals with the case of a quadratic dependence on the corresponding parameters. Further, the concept of finite BRST-antiBRST symmetry to the case of general gauge theories has been extended in Refs. [80, 81], whereas Ref. [82] by the same authors generalizes the corresponding parameters to the case of arbitrary Grassmann-odd field-dependent parameters, as compared to the so-called “potential” form of parameters [79–81].

The BV formulation and its connection to superfield/super-antifield dependent BRST symmetry have not been discussed so-far for the third quantized super-group field cosmology. This provides us a glaring omission. Here we remark that the FFBRST transformation turns to superfield/super-antifield dependent BRST transformation as the super-group field cosmology exists for super-manifold.

In the present paper we quantize the super-group field cosmology using BV-BRST method. To do so, we introduce the different gauge-fixing conditions and corresponding super-ghosts in the theory. We further demonstrate the infinitesimal BRST symmetry for the super-group field cosmology. This model of multiverse is also analysed through BV formulation where we extend the configuration space by introducing super-antifields for each set of superfields. The extended quantum actions corresponding to

the different gauge-fixing conditions are shown the different solutions of the quantum master equation. Further we generalize the BRST symmetry by making the parameter of transformation superfield/super-antifield dependent. Remarkably, we found that such generalized BRST transformation leads to the non-trivial Jacobian for the path integral measure. We explicitly compute the Jacobian for superfield/super-antifield dependent BRST transformation at general level. Furthermore, we observe that for specific choice of superfield/super-antifield dependent transformation parameter the Jacobian leads the theory from one gauge to another.

The paper is presented as follows. In section II, we discuss the supersymmetric group field cosmology in various gauges. The supersymmetric group field cosmology in BV formulation is analysed in the section III. The discussion on superfield/super-antifield dependent BRST symmetry is reported in section IV. In the last section we made a concluding remarks.

II. SUPER-GROUP FIELD COSMOLOGY AND THEIR BRST INVARIANCE

Let us start by recapitulating the progress made in [43] for the loop quantum cosmology of the spatially flat, homogeneous and isotropic universe having massless scalar field as the bulk. The loop quantum gravity is described as a gauge theory where the dynamical variables are the Ashtekar-Barbero connection K_a^i and canonical momenta, the densitized triad E_i^a . Here, $a, b = 1, 2, 3$ are the usual space index (referring to the tangent space $T_x(\Sigma)$ at x). These variables are defined in terms of co-triads e_i^a as $K_a^i(x) = K_{ab}(x)e^{bi}(x)$ and $E_i^a = |\det(e_i^a)|e_i^a(x)$, where the extrinsic curvature K_{ab} in terms of lapse N and shift N_a can be written as

$$K_{ab} = \frac{1}{2N} \left(\dot{h}_{ab} - \nabla_a N_b - \nabla_b N_a \right), \quad (1)$$

a covariant derivative on (mini-)superspace of geometries ∇_a . The four dimensional metric in this case is described by a three metric h_{ab} given as

$$h^{ab} = \delta^{ij} e_i^a e_j^b = e_i^a e_i^b, \quad (2)$$

where triad e_i^a is the inverse of co-triad e_a^i .

We construct the classical action for the super-group field cosmology, given by [42]

$$S_0 = \sum_{\nu} \int d\phi \left[D^2 \{ \Omega_i^\dagger(\varrho) \nabla_a^2 \Omega^i(\varrho) + \omega_i^a(\varrho) \omega_a^i(\varrho) \} \right]_{|}, \quad (3)$$

where “|” stands for Grassmann variable which describes a space with with supersymmetric degrees of freedom at $\theta_a = 0$ and (mini-)superspace variables $(\nu, \phi, \theta) := \varrho$. Here $\Omega(\nu, \phi, \theta)$ and $\Omega^\dagger(\nu, \phi, \theta)$ are two complex scalar super-group fields. The super-covariant derivatives of $\Omega^i(\varrho)$ and $\Omega^{i\dagger}(\varrho)$ are defined by [42]

$$\begin{aligned} \nabla_a \Omega^i(\varrho) &= D_a \Omega^i(\varrho) - i f_{kj}^i \Gamma_a^k(\varrho) \Omega^j(\varrho), \\ \nabla_a \Omega^{i\dagger}(\varrho) &= D_a \Omega^{i\dagger}(\varrho) + i f_{kj}^i \Omega^{k\dagger}(\varrho) \Gamma_a^j(\varrho), \end{aligned} \quad (4)$$

where super-derivative $D_a = \partial_a + K_a^b \theta_b$. The field-strength for a matrix valued spinor field (Γ_a^i) is given by $\omega_a^i(\varrho) = \nabla^b \nabla_a \Gamma_b^i(\varrho)$. It is evident that this classical action is invariant under the following gauge transformations:

$$\begin{aligned} \delta \Omega^i(\varrho) &= i f_{kj}^i \Lambda^k(\varrho) \Omega^j(\varrho), \\ \delta \Omega^{i\dagger}(\varrho) &= -i f_{kj}^i \Omega^{k\dagger}(\varrho) \Lambda^j(\varrho), \\ \delta \Gamma_a^i(\varrho) &= \nabla_a \Lambda^i(\varrho), \end{aligned} \quad (5)$$

where the bosonic transformation parameter Λ^i is infinitesimal in nature.

In the path integral formulation, due to this gauge symmetry there exist infinitely many $(\Lambda)\Gamma_a^i$ that are physically equivalent to Γ_a^i . This produces divergences in the functional integral. To quantize this theory consistently it is necessary to eliminate redundant gauge degrees of freedom by choosing a particular gauge. Here we choose a general gauge condition for this theory as

$$\mathcal{G}^i[\Gamma_a^i(\varrho)] = 0. \quad (6)$$

The above gauge condition can be incorporated in the theory at quantum level by adding the corresponding gauge-fixed action to the classical action. According to the Faddeev-Popov procedure the gauge-fixing condition leads to the ghost term in the effective theory. The linearised gauge-fixing action corresponding to the gauge (6) together with the induced ghost term is given by

$$S_{gf+gh} = \sum_{\nu} \int d\phi \, D^2 [B_i(\varrho) \mathcal{G}^i[\Gamma_a^i(\varrho)] + \bar{c}_i(\varrho) s_b \mathcal{G}^i[\Gamma_a^i(\varrho)]]_{|}, \quad (7)$$

where $B^i(\nu, \phi, \theta)$ is the Nakanishi-Lautrup auxiliary superfield, $c^i(\varrho)$ and $\bar{c}^i(\varrho)$ are the ghost and anti-ghost superfields respectively, and, s_b denotes the Slavnov variation. Now, the total effective action for super-group field cosmology for general gauge choice reads $S_T = S_0 + S_{gh} + S_{gf}$.

Now we consider an specific choice of Landau type gauge $\mathcal{G}^i = D^a \Gamma_a^i(\varrho) = 0$, then the total action S_T gets following identification [44]:

$$S_T = S_0 + \sum_{\nu} \int d\phi \, [D^2 \{B_i(\varrho) D^a \Gamma_a^i(\varrho) + \bar{c}_i(\varrho) D^a \nabla_a c^i(\varrho)\}]_{|}. \quad (8)$$

Furthermore, to analyse the theory in massless Curci-Ferrari type gauge (a non-linear gauge) we perform the following shift in auxiliary superfield: $B^i(\nu, \phi, \theta) \longrightarrow B^i(\varrho) - \frac{1}{2} f_{jk}^i \bar{c}^j(\varrho) c^k(\varrho)$. Performing such shift the total effective action corresponds to a non-linear gauge as follows

$$\begin{aligned} S_T = & S_0 + \sum_{\nu} \int d\phi \, [D^2 \{B_i(\varrho) D^a \Gamma_a^i(\varrho) + \bar{c}_i(\varrho) D^a \nabla_a c^i(\varrho) \\ & - \frac{1}{2} f_{jk}^i B^j(\varrho) \bar{c}_k(\varrho) c_k(\varrho) - \frac{1}{8} f_{jk}^i f_{lm}^j \bar{c}^l(\varrho) \bar{c}^m(\varrho) c_l(\varrho) c_m(\varrho)\}]_{|}. \end{aligned} \quad (9)$$

These effective actions (8) and (9) are invariant under the following (third quantized) infinitesimal BRST transformations [44]

$$\begin{aligned} \delta_b \Omega^i(\varrho) &= i f_{kj}^i c^k(\varrho) \Omega^j(\varrho) \delta\lambda, \\ \delta_b \Omega^{i\dagger}(\varrho) &= -i f_{kj}^i \Omega^{\dagger k}(\varrho) c^j(\varrho) \delta\lambda, \\ \delta_b c^i(\varrho) &= \frac{1}{2} f_{kj}^i c^k(\varrho) c^j(\varrho) \delta\lambda, \\ \delta_b \Gamma_a^i(\varrho) &= \nabla_a c^i(\varrho) \delta\lambda, \\ \delta_b \bar{c}^i(\varrho) &= B^i(\varrho) \delta\lambda, \\ \delta_b B^i(\varrho) &= 0, \end{aligned} \quad (10)$$

where $\delta\lambda$ is an infinitesimal, space-time independent anticommuting parameter. It is easy to verify that the above transformations are nilpotent of order two, i.e., $\delta_b^2 = 0$. With the help of above BRST transformation, one can write the sum of gauge-fixing and ghost parts of the action given in (7) as a BRST variation of gauge-fixed fermion $\Psi = D^2 \{\bar{c}_i(\varrho) \mathcal{G}^i[\Gamma_a^i(\varrho)]\}$ as follows

$$S_{gf+gh} = \sum_{\nu} \int d\phi \, s_b \Psi. \quad (11)$$

Such analysis will be helpful to establish theory in BV formulation.

III. SUPER-GROUP FIELD COSMOLOGY IN BATALIN-VILKOVISKY FORMULATION

To establish the theory in BV formulation we need to introduce super-antifield corresponding to each superfield having opposite statistics. In terms of superfield/super-antifield, the generating functional for the super-group field cosmology in Landau type gauge is,

$$\begin{aligned} Z_L[0] &= \int \mathcal{D}M e^{-W_L[\Phi, \Phi^*]} \\ &= \int \mathcal{D}M \exp \left[- \left(S_0 + \sum_{\nu} \int d\phi \left[\Gamma_{1i}^{a*} \nabla_a c^i + c_{1i}^* f_{kj}^i c^k c^j + \bar{c}_{1i}^* B^i \right] \right) \right], \end{aligned} \quad (12)$$

where W_L is the extended quantum action. The gauge-fixed fermion for the super-group field cosmology in Landau type gauge has the following expression:

$$\Psi_L = D^2 [\bar{c}_i(\varrho) D^a \Gamma_a^i(\varrho)]. \quad (13)$$

Now we compute the following super-antifields for corresponding to Landau gauge

$$\begin{aligned} \Omega_1^{i*} &= \frac{\delta \Psi_L}{\delta \Omega_i} = 0, \quad \Omega_1^{i\dagger*} = \frac{\delta \Psi_L}{\delta \Omega_i^\dagger} = 0, \\ c_{1i}^* &= \frac{\delta \Psi_L}{\delta c^i} = 0, \quad \bar{c}_{1i}^* = \frac{\delta \Psi_L}{\delta \bar{c}^i} = D^2 D_a \Gamma_a^i, \\ \Gamma_{1i}^{a*} &= \frac{\delta \Psi_L}{\delta \Gamma_a^i} = -D^2 D^a \bar{c}_i. \end{aligned} \quad (14)$$

However, the generating functional for the super-group field cosmology in the non-linear gauge in terms of superfields/super-antifields is given by,

$$\begin{aligned} Z_{NL}[0] &= \int \mathcal{D}M e^{-W_{NL}[\Phi, \Phi^*, \tilde{\Phi}, \tilde{\Phi}^*]} \\ &= \int \mathcal{D}M e^{-\left(S_0 + \sum_{\nu} \int d\phi \left[\Gamma_{2i}^{a*} \nabla_a c^i + c_{2i}^* \left(\frac{1}{2} f_{kj}^i c^k c^j \right) + \bar{c}_{2i}^* B^i \right] \right)}. \end{aligned} \quad (15)$$

The expression for the gauge-fixing fermion for the non-linear gauge is given by

$$\Psi_{NL} = D^2 \left[\bar{c}_i(\varrho) \left(D^a \Gamma_a^i(\varrho) - \frac{1}{4} f^{ijk} \bar{c}_j(\varrho) c_k(\varrho) \right) \right]. \quad (16)$$

The super-antifields corresponding to above gauge-fixing fermion are identified by

$$\begin{aligned} \Omega_2^{i*} &= \frac{\delta \Psi_{NL}}{\delta \Omega_i} = 0, \quad \Omega_2^{i\dagger*} = \frac{\delta \Psi_{NL}}{\delta \Omega_i^\dagger} = 0, \\ c_{2i}^* &= \frac{\delta \Psi_{NL}}{\delta c^i} = -D^2 \left(\frac{1}{4} f_i^{jk} \bar{c}_j \bar{c}_k \right), \\ \bar{c}_{2i}^* &= \frac{\delta \Psi_{NL}}{\delta \bar{c}^i} = D^2 \left[D_a \Gamma_a^i - \frac{1}{2} f_i^{jk} \bar{c}_j c_k \right], \\ \Gamma_{2i}^{a*} &= \frac{\delta \Psi_{NL}}{\delta \Gamma_a^i} = -D^2 D^a \bar{c}_i. \end{aligned} \quad (17)$$

The difference between the non-linear and linear extended quantum actions are given by

$$W_{NL} - W_L = \sum_{\nu} \int d\phi \left[c_{2i}^*(\varrho) \left(\frac{1}{2} f_{kj}^i c^k(\varrho) c^j(\varrho) \right) + (\bar{c}_{2i}^*(\varrho) - \bar{c}_{1i}^*(\varrho)) B^i(\varrho) \right]. \quad (18)$$

Here we note that these extended quantum actions, $W_\Psi[\Phi, \Phi^*] \equiv (W_{NL}, W_L)$, are solutions of the following mathematically rich relation so-called quantum master equation,

$$\Delta e^{iW_\Psi[\Phi, \Phi^*]} = 0, \quad \Delta \equiv (-1)^\epsilon \frac{\partial_l}{\partial \Phi} \frac{\partial_l}{\partial \Phi^*}. \quad (19)$$

In the next section, we shall establish a map between the two generating functionals corresponding to the above extended actions using the technique of superfield/super-antifield dependent BRST transformations.

IV. GENERALIZED BRST SYMMETRY FOR SUPER-GROUP FIELD COSMOLOGY

In this section, we analyse the superfield/super-antifield dependent BRST transformation which is characterized by the superfield/super-antifield dependent BRST parameter. For this purpose, we first write the usual BRST transformation given in (10) in compact form as following:

$$\Phi'_\alpha(x) - \Phi_\alpha(x) = \delta_b \Phi_\alpha(x) = s_b \Phi_\alpha(x) \delta\lambda = \mathcal{R}_\alpha(x) \delta\lambda, \quad (20)$$

where $\mathcal{R}_\alpha(x)(s_b \Phi_\alpha(x))$ is the Slavnov variations of the collective superfield $\Phi_\alpha(x)$ satisfying $\delta_b \mathcal{R}_\alpha(x) = 0$. Here the infinitesimal transformation parameter $\delta\lambda$ is a Grassmann parameter and doesn't depend on any superfield/super-antifield.

Now, we propose the superfield/super-antifield dependent BRST transformation as follows

$$\delta_b \Phi_\alpha(x) = \Phi'_\alpha(x) - \Phi_\alpha(x) = \mathcal{R}_\alpha(x) \Lambda[\Phi, \Phi^*], \quad (21)$$

where the Grassmann parameter $\delta\lambda$ is replaced by $\Lambda[\Phi, \Phi^*]$ which depends on the superfield/super-antifield explicitly. The novelty of superfield/super-antifield dependent BRST transformation is that although being symmetry of the extended action such transformation does not leave the functional measure invariant and leads a non-trivial local Jacobian.

Now, we evaluate the Jacobian of functional measure under superfield/super-antifield dependent BRST transformation (within functional integral) as follows

$$\begin{aligned} Z'_L[0] &= \int \mathcal{D}M (\text{sDet} J[\Phi, \Phi^*]) \exp\{-W_L[\Phi, \Phi^*]\}, \\ &= \int \mathcal{D}M e^{-(W_L[\Phi, \Phi^*] - i \text{sTr} \ln J[\Phi, \Phi^*])}, \end{aligned} \quad (22)$$

where Z'_L denotes the generating functional under change of variables. The Jacobian matrix for the superfield/super-antifield dependent BRST transformation is computed by

$$J_\alpha^\beta[\Phi, \Phi^*] = \frac{(\delta \Phi'_\alpha, \delta \tilde{\Phi}'_\alpha)}{(\delta \Phi_\beta, \delta \tilde{\Phi}_\beta)} = \delta_\alpha^\beta + \frac{\delta \mathcal{R}_\alpha(x)}{\delta \Phi_\beta} \Lambda[\Phi, \Phi^*] + \mathcal{R}_\alpha(x) \frac{\delta \Lambda[\Phi, \Phi^*]}{\delta \Phi_\beta}. \quad (23)$$

The nilpotency property of the BRST transformation (i.e. $s_b^2 = 0$) and relation (23) yield

$$\text{sTr} \ln J[\Phi, \Phi^*] = -\ln(1 + s_b \Lambda[\Phi, \Phi^*]), \quad (24)$$

which simplifies to

$$\text{sDet} J[\Phi, \Phi^*] = \frac{1}{1 + s_b \Lambda[\Phi, \Phi^*]}. \quad (25)$$

With this Jacobian the relation (22) modified by

$$Z'_L[0] = \int \mathcal{D}\Phi \exp \left(-W_L[\Phi, \Phi^*] - i \ln(1 + s_b \Lambda[\Phi, \Phi^*]) \right). \quad (26)$$

This means that the effective quantum action gets change under the superfield/super-antifield dependent BRST transformation characterized by an arbitrary $\Lambda[\Phi, \Phi^*]$. Now we compute the specific change in the effective action of the super-group field cosmology under the superfield/super-antifield dependent BRST transformation having an specific $\Lambda[\Phi, \Phi^*]$. Therefore, we construct the specific parameter as follows

$$\Lambda[\Phi, \Phi^*] = \sum_{\nu} \int d\phi \bar{c}^l B_l (B^2)^{-1} \left(\exp \left\{ -i \left[c_{2i}^*(\varrho) \left(\frac{1}{2} f_{kj}^i c^k(\varrho) c^j(\varrho) \right) + (\bar{c}_{2i}^*(\varrho) - \bar{c}_{1i}^*(\varrho)) B^i(\varrho) \right] \right\} - 1 \right), \quad (27)$$

where $B^2 =: B^i B_i$. Now the Slavnov variation gives

$$s_b \Lambda[\Phi, \Phi^*] = \sum_{\nu} \int d\phi \exp \left(-i \left[c_{2i}^*(\varrho) \left(\frac{1}{2} f_{kj}^i c^k(\varrho) c^j(\varrho) \right) + (\bar{c}_{2i}^*(\varrho) - \bar{c}_{1i}^*(\varrho)) B^i(\varrho) \right] \right) - 1. \quad (28)$$

The Jacobian (24) for the parameter (27) gets the following identification:

$$i \ln(1 + s_b \Lambda[\Phi, \Phi^*]) = \sum_{\nu} \int d\phi \left[c_{2i}^*(\varrho) \left(\frac{1}{2} f_{kj}^i c^k(\varrho) c^j(\varrho) \right) + (\bar{c}_{2i}^*(\varrho) - \bar{c}_{1i}^*(\varrho)) B^i(\varrho) \right]. \quad (29)$$

Therefore, from the expressions (18), (26) and (29) it is evident that

$$Z'_L[0] = Z_{NL}[0]. \quad (30)$$

Hence, it is observed that the superfield/super-antifield dependent BRST transformation with parameter (27) correlate two different solutions of the quantum master equation.

V. CONCLUDING REMARKS

In this paper we consider the supersymmetric group field cosmology, a model for homogeneous and isotropic multiverse. In the multiverse scenario, the gauge and the matter sectors describe the different universes. We study the supersymmetrization of group field cosmology which is a gauge invariant theory. The third quantized infinitesimal BRST transformations for the super-group field cosmology is demonstrated for the Landau type and Curci-Ferrari type gauge-fixing conditions. Further, the super-group field cosmology is studied in the context of BV formulation. This may provide a consistent quantum description of supersymmetric group field cosmology. In this approach we introduce the super-antifield corresponding to each superfield of the theory. The extended actions of the super-group field cosmology are shown the solutions of the mathematically rich quantum master equation. The quantum master equation is an important identities for such model.

Furthermore, we have generalized the BRST transformation of the super-group field cosmology by making the transformation parameter superfield/super-antifield dependent. We have found that under the infinitesimal BRST transformation both the effective action and the functional measure remain invariant. However, under superfield/super-antifield dependent BRST transformation only effective action remains invariant while the functional measure does not. We have computed the Jacobian for functional measure under arbitrary superfield/super-antifield dependent BRST transformation explicitly. Remarkably, we have found that under specific superfield/super-antifield dependent BRST transformation the Jacobian switches the void functional from Landau type gauge-fixing condition to Curci-Ferrari type gauge-fixing condition. Such analysis may be an important towards the establishment of the quantum theory of super-

group field cosmology. Further implications of superfield/super-antifield dependent BRST transformation on the multiverse model, for example calculating the certain observables, will be subject of interest.

-
- [1] D. Oriti and L. Sindoni, *New J. Phys.* 13, 025006 (2011).
 - [2] C. Rovelli, *Quantum Gravity* (Cambridge University Press, Cambridge, U.K., 2007).
 - [3] T. Thiemann, *Modern Canonical Quantum General Relativity* (Cambridge University Press, Cambridge, U.K., 2007).
 - [4] A. Ashtekar, *Phys. Rev. D* 36, 1587 (1987).
 - [5] A. Ashtekar, *Phys. Rev. Lett.* 57, 2244 (1986).
 - [6] A. Ashtekar and A. Corichi, *Class. Quantum Grav.* 20, 4473 (2003).
 - [7] A. Ashtekar, *Nature Phys.* 2, 726 (2006).
 - [8] E. Livine and J. Tambornino, *J. Math. Phys.* 53, 012503 (2012).
 - [9] M. Geiller M, M. L. Rey and K. Noui *Phys. Rev. D* 84, 044002 (2011).
 - [10] M. Bojowald, *Living Rev. Relativity* 8, 11 (2005).
 - [11] A. Ashtekar, M. Bojowald and L. Lewandowski, *Adv. Theor. Math. Phys.* 7, 233 (2003).
 - [12] L. Freidel, *Int. J. Theor. Phys.* 44, 1769 (2005).
 - [13] D. Oriti, The microscopic dynamics of quantum space as a group field theory, in: *Foundations of space and time*, G. Ellis, J. Murugan (eds.). (Cambridge University Press, Cambridge, U.K., 2011).
 - [14] D. Oriti, in *Quantum Gravity*, edited by B. Fauser et al. (Birkhaeuser, Basel, Switzerland, 2008) [gr-qc/0512103].
 - [15] D. Oriti, in *Foundations of Space and Time: Reflections on Quantum Gravity*, edited by J. Murugan et al. (Cambridge University Press, Cambridge, U.K., 2012) [arXiv:1110.5606].
 - [16] D. Oriti, *Rep. Prog. Phys.* 64, 1489 (2001).
 - [17] A. Perez, *Class. Quantum Grav.* 20, R43 (2003).
 - [18] A. Buonanno, M. Gasperini, M. Maggiore and C. Ungarelli, *Class. Quantum Grav.* 14, L97 (1997).
 - [19] L. O. Pimentel and C. Mora, *Phys. Lett. A* 280, 191 (2001).
 - [20] S. R. Perez and P. F. G. Diaz, *Phys. Rev. D* 81, 083529 (2010).
 - [21] V. P. Maslov and O. Yu. Shvedov, *Phys. Rev. D* 60, 105012 (1999).
 - [22] P. Ivanov and S. V. Chernov, *Phys. Rev. D* 92, 063507 (2015).
 - [23] S. P. Kim, arXiv:1212.5355 [gr-qc].
 - [24] M. Faizal, *JETP* 114, 400 (2012); *Int. J. Geom. Meth. Mod. Phys.* 11, 1450010 (2014); *Mod. Phys. Lett. A* 27, 1250007 (2012); *Int. J. Mod. Phys. A* 30, 1550036 (2015).
 - [25] Y. Ohkuwa, M. Faizal and Y. Ezawa, *Annals. Phys.* 365, 54 (2016).
 - [26] A. Strominger, Baby universes, in *Quantum Cosmology and Baby Universes*, Vol. 7, ed. by S. Coleman, J. B. Hartle, T. Piran and S. Weinberg, World Scientific, London (1990).
 - [27] A. Baratin and D. Oriti, *Phys. Rev. D* 85, 044003 (2012).
 - [28] M. Smerlak, *Class. Quantum Grav.* 28, 178001 (2011).
 - [29] A. Baratin, F. Girelli and D. Oriti, *Phys. Rev. D* 83, 104051 (2011).
 - [30] A. Tanasa, *J. Phys. A: Math. Theor.* 45, 165401 (2012).
 - [31] A. Ashtekar, *J. Phys. Conf. Ser.* 189, 012003 (2009).
 - [32] L. Qin, G. Deng and Y. Ma, *Commun. Theor. Phys.* 57, 326 (2012).
 - [33] B. Gupta and P. Singh, *Phys. Rev. D* 85, 044011 (2012).
 - [34] E. W. Ewing, *Class. Quant. Grav.* 29, 085005 (2012).
 - [35] L. Qin and Y. Ma, *Mod. Phys. Lett. A* 27, 1250078 (2012).
 - [36] G. Calcagni, S. Gielen and D. Oriti, *Class. Quantum Grav.* 29, 105005 (2012).
 - [37] L. A. Glinka, *Grav. Cosmol.* 15, 317 (2009).
 - [38] S. Gielen, *J. Phys. Conf. Ser.* 360, 012029 (2012).
 - [39] P. van Nieuwenhuizen, *Phys. Rep.* 68, 189 (1981).
 - [40] D. Freedman and P. van Nieuwenhuizen, *Scientific American* 282, 126 (1978).
 - [41] S. Akula, M. Liu, P. Nath and G. Peim *Phys. Lett. B* 709, 192 (2012).
 - [42] M. Faizal, *Class. Quant. Grav.* 29, 215009 (2012).
 - [43] S. Upadhyay, *Gen. Rel. Grav.* 46, 1678 (2014); *Mod. Phys. Lett. A* 30, 1550072 (2015).
 - [44] M. Faizal, *Grav. Cosmol.* 20, 2 (2014).
 - [45] D. Binosi and A. Quadri, arXiv:1511.09309 [hep-th].
 - [46] J. Francois, S. Lazzarini and T. Masson, arXiv:1508.07666 [math-ph].

- [47] M. Faizal, Found. Phys. 41, 270 (2011); J. Phys. A 44, 402001 (2011); Phys. Lett. B 705, 120 (2011); Phys. Rev. D 84, 106011 (2011); Commun. Theor. Phys. 57, 637 (2012); Mod. Phys. Lett. A 27, 1250075 (2012); Comm. Theor. Phys. 58, 704 (2012); Int. J. Theor. Phys. 52, 392 (2013); JHEP 1301, 156 (2013); Int. J. Mod. Phys. A 28, 1350012 (2013); Mod. Phys. Lett. A 28, 1350034 (2013); Phys. Lett. B 727, 536 (2013); Comm. Theor. Phys. 62, 697 (2014); P. Weinreb and M. Faizal, Phys. Lett. B 748, 102 (2015); M. Faizal and D. J. Smith, Phys. Rev. D 85, 105007 (2012); M. Faizal and M. Khan, Eur. Phys. J. C 71, 1603 (2011).
- [48] M. Henneaux and C. Teitelboim, *Quantization of gauge systems*. (Princeton, USA: Univ. Press, 1992).
- [49] S. Weinberg, *The quantum theory of fields, Vol-II: Modern applications*. (Cambridge, UK Univ. Press, 1996).
- [50] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B 102, 27 (1981).
- [51] I. A. Batalin and G. A. Vilkovisky, Phys. Rev. D 28, 2567 (1983); D 30, 508 (1984) (E).
- [52] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B 120, 166 (1983).
- [53] W. Siegel, Phys. Lett. B 149, 157 (1984); Phys. Lett. B 149, 162 (1984).
- [54] A. A. Sharapov, arXiv: 1511.05711 [math-ph].
- [55] R. R. Metsaev, arXiv:1508.07928 [hep-th].
- [56] S. D. Joglekar and B. P. Mandal, Phys. Rev. D 51, 1919 (1995).
- [57] S. D. Joglekar and B. P. Mandal, Int. J. Mod. Phys. A 17, 1279 (2002).
- [58] S. Upadhyay, S. K. Rai and B. P. Mandal, J. Math. Phys. 52, 022301 (2011).
- [59] S. Upadhyay and B. P. Mandal, Phys. Lett. B 744, 231 (2015); Int. J. Theor. Phys. 55, 1 (2016); Eur. Phys. J. C 72, 2065 (2012); Annals of Physics 327, 2885 (2012); Eur. Phys. Lett. 93, 31001 (2011); Mod. Phys. Lett. A 25, 3347 (2010).
- [60] S. Upadhyay, M. K. Dwivedi and B. P. Mandal, Int. J. Mod. Phys. A 30, 1550178 (2015); Int. J. Mod. Phys. A 28, 1350033 (2013).
- [61] M. Faizal, B. P. Mandal and S. Upadhyay, Phys. Lett. B 721, 159 (2013).
- [62] S. Upadhyay and P. A. Ganai, arXiv: 1605.04290.
- [63] S. Upadhyay, A. Reshetnyak and B. P. Mandal, arXiv:1605.02973 [physics.gen-ph].
- [64] B. P. Mandal, S. K. Rai and S. Upadhyay, Eur. Phys. Lett. 92, 21001 (2010).
- [65] S. Upadhyay and D. Das, Phys. Lett. B 733, 63 (2014).
- [66] S. Upadhyay, Phys. Lett. B 727, 293 (2013).
- [67] S. Upadhyay, Annals. Phys. 340, 110 (2014).
- [68] S. Upadhyay and B. P. Mandal, Eur. Phys. J. C 75, 327 (2015).
- [69] S. Upadhyay, Annals. Phys. 344, 290 (2014).
- [70] S. Upadhyay and B. P. Mandal, Prog. Theor. Exp. Phys. 053B04 (2014).
- [71] S. Upadhyay, Annals. Phys. 356, 299 (2015); EPL 104, 61001 (2013); EPL 105, 21001 (2014).
- [72] R. Banerjee, B. Paul and S. Upadhyay, Phys. Rev. D 88, 065019 (2013).
- [73] R. Banerjee and S. Upadhyay, Phys. Lett. B 734, 369 (2014).
- [74] M. Faizal, S. Upadhyay and B. P. Mandal, Phys. Lett. B 738, 201 (2014); Int. J. Mod. Phys. A 30, 1550032 (2015); Eur. Phys. J. C 76, 189 (2016).
- [75] S. Upadhyay, Phys. Lett. B 740, 341 (2015).
- [76] S. Upadhyay, M. Oksanen and R. Bufalo, arXiv:1510.00188 [hep-th].
- [77] S. Upadhyay and B. Paul, arXiv:1506.0525.
- [78] S. Upadhyay, Annals. Phys. 356, 299 (2015).
- [79] P. Y. Moshin and A. A. Reshetnyak, Nucl. Phys. B 888, 92 (2014).
- [80] P. Y. Moshin and A. A. Reshetnyak, Int. J. Mod. Phys. A 30, 1550021 (2015).
- [81] P. Y. Moshin and A. A. Reshetnyak, Phys. Lett. B 739, 110 (2014).
- [82] P. Y. Moshin and A. A. Reshetnyak, arXiv: 1506.04660[hep-th].